Computer-Aided Synthesis of a Lossy Commensurate Line Network and Its Application in MMIC's

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Abstract —**In this paper, a useful theorem which extends a previously introduced lossy transformation technique [1] to more general applications is proposed for transformation between distributed lossy and lumped lossless networks, and a corollary is given for extension of the well-known Kuroda identities to the general lossy case. A new computer-aided approach is developed for the synthesis of lossy commensurate line networks with all lines having arbitrary frequency-dependent losses. As an application, two broad-band amplifiers are designed for monolithic microwave integrated circuits (MMIC's) and their performances are compared with the examples in [2] and [3].**

I. Introduction

It is well known that MMIC's have found a variety of applications in radar, spaceflight, satellites, and military communications. Matching networks are the most important parts of MMIC's and are usually constructed from lumped and distributed elements. These matching elements, which are fabricated on semi-insulating GaAs substrates, have losses much greater than those of matching elements developed for conventional hybrid integrated circuits. Thus, existing techniques that are considered efficient [2]–[10] for synthesizing lumped and distributed lossless matching networks are not suitable for lossy matching networks. To solve this problem, a theorem has been introduced [1] for transformation between lumped lossy and lossless networks, and a new computer-aided lossy transformation technique has been developed for treating the synthesis of lumped matching networks with arbitrary nonuniform losses. However, it is clear that only a small part of the problem has been solved. As frequency increases to the millimeter-wave region, the distributed network is superior to the lumped one in many aspects. For example, transmission lines have lower parasitic reactances and their characteristic impedances can be realized easily and exactly.

The modern design of microwave TEM distributed networks is based upon a complex plane transformation introduced by Richards in 1948 [11]. Later, other authors were stimulated by his article and significant achievements were made. The well-known Kuroda identities were among these contributions, making it possible to realize synthesized commensurate line networks. As for the synthesis of distributed lossy network, to our knowledge, few published papers have dealt with it.

In order to consider the losses of transmission lines in the synthesis of a distributed network, a new and useful theorem is introduced which extends the lossy transformation technique described in [1] to wider applications and makes it possible to obtain the parameters of a corresponding lossy commensurate line network from those of a lumped lossless network. The well-known Kuroda identities are extended by a corollary to the general lossy commensurate line network. It will be seen by comparison with the results shown in [2] and [3] that the new method is practically applicable and considerably simplified, and is able to yield any complex models of the commensurate lines with arbitrary frequency-dependent losses. Furthermore, a computer-aided procedure is presented to show the detailed synthesis steps of two broad-band monolithic microwave integrated FET amplifiers with lossy commensurate line networks as matching networks.

II. Transformation and Kuroda's Identities of the Lossy Commensurate Line Network

Richards, in his famous paper [11], first used the following transformation:

$$\lambda = \tanh[\gamma(s)l] \quad (1)$$

to synthesize a lossless commensurate line network,[1] where $\gamma(s)$ is the propagation constant; $s = j\omega$, the complex angular frequency; and l is the length of all the transmission line elements. By this relation, a complex angular frequency in the s plane will be mapped into the λ plane, and many theorems for lumped lossless networks can be "translated" into theorems on the lossless commensurate line network. Thereafter, the Richards transformation became the theoretical basis for almost all the published papers on distributed network synthesis, and the relevant theorems were used to treat the lossless commensurate line network. However, a general method had not been found for the synthesis of a lossy commensurate line network with all lines having arbitrary frequency-dependent losses.

In general, the propagation constant and characteristic impedance of a lossy transmission line are frequency-depen-

[1]Small losses were considered under the assumption that all lines in the circuit are distortionless. But the change of frequency variable must be made in the true frequency plane, as was done by Darlington [12].

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dent functions and can be written as

$$\gamma(s)=\sqrt{(sL+R)(sC+G)}=\beta_0\gamma_0(s) \tag{2a}$$

$$Z_0(s)=\sqrt{(sL+R)/(sC+G)}=Z_{0,t}\delta(s) \tag{2b}$$

in which

$$\beta_0=\omega_m/C_{vp} \tag{3a}$$

$$\gamma_0(s)=\sqrt{(j\omega_n+1/Q_l)(j\omega_n+1/Q_c)} \tag{3b}$$

$$Z_{0,t}=\sqrt{L/C} \tag{3c}$$

$$\delta(s)=\sqrt{(j\omega_n+1/Q_l)/(j\omega_n+1/Q_c)} \tag{3d}$$

where R, L, G, and C are the series resistance, series inductance, shunt conductance, and shunt capacitance, all per unit length, for a given line. $Q_l=\omega_m L/R$ and $Q_c=\omega_m C/G$ are the quality factors of the conductor and dielectric of the line, respectively, at measured angular frequency ω_m. $C_{vp}=1/\sqrt{LC}$ is the velocity of propagation on the line, and $\omega_n=\omega/\omega_m$ is the normalized angular frequency. From (2b), (3c), and (3d), it can be found that the characteristic impedance, $Z_0(s)$, may be divided into two parts, one being the frequency-dependent function $\delta(s)$, and the other the real positive multiplicative constant, $Z_{0,t}$. If the lossy transmission line reduces to a corresponding lossless one, $\delta(s)$ will be equal to 1 and $Z_0(s)$ to $Z_{0,t}$, which is usually called the static characteristic impedance of the lossless transmission line.

Now, assume that all elements constructed by the lossy transmission lines have the same $\gamma(s)$ and $\delta(s)$. Then the impedances of short- and open-circuited stubs will have the following expressions:

$$Z_{sh}(s)=Z_{0,sh}\delta(s)\tanh[\gamma(s)l] \tag{4a}$$

$$Z_{op}(s)=Z_{0,op}\delta(s)/\tanh[\gamma(s)l] \tag{4b}$$

where $Z_{0,sh}$ and $Z_{0,op}$ are similar to $Z_{0,t}$, and $Z_{0,sh}\delta(s)$ and $Z_{0,op}\delta(s)$ are frequency-dependent characteristic impedances of the short- and open-circuited stubs.

If a lossy commensurate line network contains only the short- and open-circuited stubs, then by substituting

$$Z_1=\delta(s)\tanh[\gamma(s)l] \tag{5a}$$

$$Z_2=\delta(s)/\tanh[\gamma(s)l] \tag{5b}$$

in (4), we have

$$Z_{sh}(s)=Z_{0,sh}Z_1 \tag{6a}$$

$$Z_{op}(s)=Z_{0,op}Z_2. \tag{6b}$$

Thus, in terms of the transformation introduced in [1], the lossy commensurate line network can be transformed to a corresponding lumped lossless network or the Richards transformation can be used to transform the short- and open-circuited stubs to lossless inductors and capacitors, respectively, if the stubs are lossless.

But a lossy commensurate line network without a finite lossy transmission line, which is usually incorporated as a distributed lossy *unit element* (UE), will be practically useless. However, unlike a finite lossless transmission line, the distributed lossy UE cannot be transformed directly by the Richards correspondence. Furthermore, it is uncertain whether or not the distributed lossy UE can be transformed as the short- and open-circuited stubs. In order to solve this problem, the transformation in [1] is revised and a new theorem, given below, is proposed which can be employed to transform the distributed lossy UE to a corresponding "lumped lossless UE."

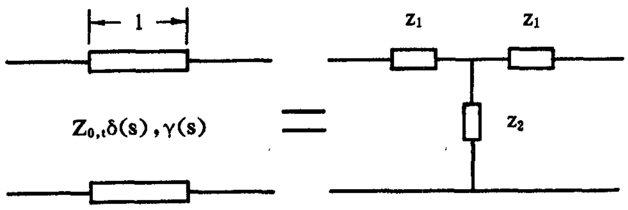

Fig. 1. Equivalent circuit of a distributed lossy unit element.

Theorem: If each element of a basic unit or of its equivalent circuit produces an individual impedance equal to the product of Z_2 and a rational function of Z_1/Z_2, except for a possible branch point at $Z_1/Z_2=1$, where Z_1 and Z_2 are any physically realizable impedances, then the impedance matrix of any lossy or lossless network N constructed by these basic units, if it exists, can be transformed to the impedance matrix of a corresponding lossless network M. That is,

$$\tilde{Z}(\lambda)=Z/\sqrt{Z_1Z_2}=F(\lambda^2)/\lambda \tag{7a}$$

$$Z(s)=\tilde{Z}\sqrt{Z_1Z_2}=Z_2F(Z_1/Z_2) \tag{7b}$$

where $F(x)$ is a matrix with its elements being the rational functions of x. $\tilde{Z}(\lambda)$ and $Z(s)$ are impedance matrices of M and N, respectively, with λ and s being their complex angular frequencies, which are related by

$$\lambda=\sqrt{Z_1(s)/Z_2(s)}. \tag{8}$$

(See the proof in the Appendix.)

It can be easily verified that (8) maps both right halves of the s and λ planes to each other. This mapping is certainly not one-to-one, but the multiple-valued state of the inverse corresponds merely to the periodicity of $Z(s)$ given by (7b).

It should be emphasized that although this theorem is based upon the theorem in [1] and has a form of transformation similar to [1, corollary 1] the conditions between them are different. The former extends the original condition of the latter to a more general case. That is, all that is required is that each element in the impedance matrix or in the equivalent circuit of a basic unit have the form $Z_2f(Z_1/Z_2)$. Of course, this form can reduce to Z_1 or Z_2. Therefore, this theorem is especially suitable for those elements which cannot simply be represented by the impedance which is proportional to Z_1 or Z_2. The advantage of the theorem can be seen clearly by the following example.

Example

A distributed lossy UE with line length l (shown in Fig. 1) can be considered as a basic unit and is generally expressed by its transfer matrix

$$T=\begin{bmatrix}\cosh[\gamma(s)l] & Z_{0,t}\delta(s)\sinh[\gamma(s)l]\\ \sinh[\gamma(s)l]/[Z_{0,t}\delta(s)] & \cosh[\gamma(s)l]\end{bmatrix}. \tag{9}$$

Then it can be found by substituting (5) in the elements, z_1 and z_2, of its equivalent circuit that the condition of the theorem is satisfied, i.e.,

$$z_1 = Z_{0,t} Z_2 \left[1 - \sqrt{1 - Z_1/Z_2}\right] \tag{10a}$$

$$z_2 = Z_{0,t} Z_2 \sqrt{1 - Z_1/Z_2}. \tag{10b}$$

Here, $Z_{0,t}$, $\delta(s)$, and $\gamma(s)$ are given by (2) and (3). Thus, with the help of (7), $\tilde{Z}$, the impedance matrix of a corresponding "lumped lossless UE," can be obtained from Z, the impedance matrix of the distributed lossy UE:

$$\tilde{Z} = \frac{Z}{\sqrt{Z_1 Z_2}} = \frac{1}{\sqrt{Z_1 Z_2}} \begin{bmatrix} z_1 + z_2 & z_2 \\ z_2 & z_1 + z_2 \end{bmatrix} = \frac{Z_{0,t}}{\lambda} \begin{bmatrix} 1 & \sqrt{1-\lambda^2} \\ \sqrt{1-\lambda^2} & 1 \end{bmatrix} \tag{11}$$

where $\lambda = \sqrt{Z_1/Z_2} = \tanh[\gamma(s)l]$.

Since a lossy commensurate line network is usually constructed by the distributed lossy UE's and by short- and open-circuited lossy stubs, an important conclusion can be drawn. This is that if all the elements of the lossy commensurate line network have a common propagation constant, $\gamma(s)$, and their characteristic impedances are all proportional to $\delta(s)$, where in addition to the expressions given in (2) and (3), $\gamma(s)$ and $\delta(s)$ may be any other frequency-dependent functions, there will be a one-to-one transformation between the lossy commensurate line network and its corresponding lumped lossless network with distributed lossy UE's, short-circuited lossy stubs, and open-circuited lossy stubs corresponding to "lumped lossless UE's," lossless inductors, and capacitors, respectively.

Therefore, by means of the following transformation [1]:

$$\tilde{S}(\lambda) = \left\{(I+S) - \sqrt{Z_1 Z_2}(I-S)\right\} \cdot \left\{(I+S) + \sqrt{Z_1 Z_2}(I-S)\right\}^{-1} \tag{12a}$$

$$S(s) = \left\{\sqrt{Z_1 Z_2}(I+\tilde{S}) - (I-\tilde{S})\right\} \cdot \left\{\sqrt{Z_1 Z_2}(I+\tilde{S}) + (I-\tilde{S})\right\}^{-1} \tag{12b}$$

where $S(s)$ and $\tilde{S}(\lambda)$ are the unit normalized scattering matrices of any lossy or lossless network N and its corresponding lossless network M, respectively, I is the identity matrix, and Z_1 and Z_2 are as defined in (5), the unknown unit normalized scattering parameters of the lossy commensurate line network can be obtained from those of a previously assumed lumped lossless network. A detailed description will be given in Section III.

It should be noted that although the λ in (11) has the same expression as that employed by Richards, the transformation given in (12) cannot be achieved by directly using his correspondence. In reverse, it can be seen by carefully considering the transformation (12) that the Richards transformation may be considered as a special case of our transformation. For example, if all of the lossy lines in a commensurate line network reduce to their corresponding lossless ones, $\delta(s)$ will be equal to 1 and $\gamma(s)$ to $s\tau$, where $\tau = l/C_{vp}$. These will result in $\sqrt{Z_1 Z_2} = 1$ and $\lambda = \sqrt{Z_1/Z_2} = \tanh(s\tau)$. In this case, (12) will become the well-known Richards correspondence, i.e.,

$$S(s) = \tilde{S}(\lambda)|_{\lambda = \tanh(s\tau)} \tag{13}$$

where $s = j\omega$.

Even though the lossy commensurate line network may be synthesized by means of the transformation mentioned above, without the corresponding Kuroda identities, the synthesized network may sometimes be impracticable.[2] Are the Kuroda identities still valuable in this lossy case? The answer is yes!

Corollary: The Kuroda identities, which are suitable for the lossless commensurate line network, will still hold for the corresponding lossy commensurate line network.

In applying the corollary, one point to which attention should be paid, is that because of the lossy property of the distributed lossy UE, a new distributed lossy UE introduced in front of the 1 Ω source or load resistor and shifted into place using Kuroda's identities for making the synthesized network practically realizable, as is usually done for lossless transmission lines [9], will cause the characteristics of the final network to deviate from those of the originally synthesized one. Certainly, the larger the number of distributed lossy UE's introduced, the larger the resulting deviation. To solve this problem, two methods may be employed. In the first, a proper ratio between the number of distributed lossy **UE's and lossy stubs is chosen. Thus, the series and short-circuited stubs, which are difficult to realize in practice, can be transformed by shifting the relevant distributed lossy UE's contained in the network. In the second method, the deviation resulting from the introduced distributed lossy UE's is adjusted by optimizing the final network.**

III. Application of the Distributed Lossy Transformation Technique

In order to clearly demonstrate the synthesis procedure of the lossy commensurate line network and its application in MMIC's, the detailed synthesis steps for two broad-band monolithic microwave integrated FET amplifiers are illustrated.

Example 1

Step 1: The lumped lossless networks, which can be realized by ideal transformers, "lumped lossless UE's," and lossless inductors and capacitors, are assumed to correspond to lossy commensurate line matching networks and to have the following forms of unit normalized scattering parameters:

$$\tilde{e}_{11,v}(\lambda) = \frac{h(\lambda)}{g(\lambda)} = \frac{h_1 + h_2\lambda + h_3\lambda^2 + \cdots + h_{n+1}\lambda^n}{g_1 + g_2\lambda + g_3\lambda^2 + \cdots + g_{n+1}\lambda^n} \tag{15a}$$

$$\tilde{e}_{12,v}(\lambda) = \tilde{e}_{21,v}(\lambda) = f(\lambda)/g(\lambda) = (+/-)\lambda^k(1-\lambda^2)^{m/2}/g(\lambda) \tag{15b}$$

$$\tilde{e}_{22,v}(\lambda) = (-1)^{k+1} h(-\lambda)/g(\lambda) \qquad (\boldsymbol{v = 1, 2, \ldots, NM}). \tag{15c}$$

Here NM is the number of networks; $h(\lambda)$ and $g(\lambda)$ are the numerator and denominator polynomials of $\tilde{e}_{11,v}(\lambda)$ and have the same degree n. The numerator polynomial $f(\lambda)$ of

[2] Even if the low-pass Kuroda identities are the same as the Richards technique by alternately removing a shunt open-circuited stub followed by a cascaded line, and high-pass Kuroda identities are equivalent to the partial stub extraction [14], the Kuroda identities are somewhat more convenient for practical applications.

$\tilde{e}_{12,v}(\lambda)$ has degree $m+k \leqslant n$, where m and k represent the number of lumped lossless UE's and high-pass elements, respectively. Thus, the number of low-pass elements is determined by $n-(m+k)$.

It should be pointed out that even though the number of total matching elements is specified by n, the optimal number of elements will be determined by adjusting the coefficient h_i $(i=1,2,\cdots,n+1)$ of $h(s)$ in an optimization procedure, and the coefficient g_i of $g(s)$ will be correspondingly specified in accordance with the unitary property of a lossless network [7], [13]. Also, since the line length l_v $(v=1,2,\cdots,NM)$ in our technique is used as a variable for obtaining even better performance, its value, which is chosen as a quarter wavelength at 1.5 times the high-frequency limit of the passband, will become an initial value in the optimization. Thus, the final line lengths of the matching elements in a matching network may differ from those in another matching network.

Step 2: Q_l and Q_c of the distributed lossy UE's and lossy stubs are assumed to be 80 and 120 at f_m = 14 GHz and $C_{vp} = 3\times 10^{11}$ mm/s, and β_0, $\gamma(s)$, and $\delta(s)$ are then computed. By substituting (3a) and (3b) into (2a), and (2a) and (3d) into (5), respectively, Z_1 and Z_2 in (12) can be calculated. Thus, the unit normalized scattering parameters, $e_{ij,v}(s)$ $(i,j=1,2)$, of the lossy commensurate line matching networks can be obtained from (15) via (12).

Step 3: The numerically specified scattering parameters of a HP 1 μm MESFET are used [2] across the octave band of 7–14 GHz. The source and load impedances of the amplifier, Z_g and Z_l, are specified to be 50 Ω. The expression for the *transducer power gain* (TPG) of a lossy matched amplifier [1] is employed in the optimization. It can be found by analyzing the FET's scattering parameters that the device is absolutely stable with the calculated maximum available gain from 14.73 dB at 7 GHz to 7.98 dB at 14 GHz. Thus, from the maximum gain–bandwidth point of view, the goal of a flat gain level, $T_0(\omega)$, to be approached by the TPG should not exceed 7.98 dB. Therefore, considering the losses in the lossy matching elements and the calculated maximum available gain at 14 GHz, $T_0(\omega)$ is specified to be 7.0 dB over the octave band.

Step 4: With h_i being unknown variables, a better TPG will be achieved by an optimization routine, and $\tilde{e}_{11,v}(\lambda)$ in (15a) can then be determined. Thus, the lumped lossless network can be realized by first applying Richards' theorem m times to the input impedance $[1+\tilde{e}_{11,v}(\lambda)]/[1-\tilde{e}_{11,v}(\lambda)]$ to extract m cascaded lumped lossless UE's, corresponding to the term $(1-\lambda^2)^{m/2}$ in (15b), and then extracting a ladder network of series or shunt lossless inductors or capacitors in the λ domain. Afterwards, the topology of the lossy commensurate line network can be easily obtained by substituting distributed lossy UE's and short- and open-circuited stubs for the corresponding lumped lossless UE's, inductors, and capacitors, respectively. For this example, the assumed lumped lossless networks for input and output matches are computed as

$$\tilde{e}_{11,1}(\lambda)=\frac{-0.6643\lambda-0.5439\lambda^2+0.6076\lambda^3}{1+2.6569\lambda+2.3091\lambda^2+0.6076\lambda^3}$$

$$\tilde{e}_{11,2}(\lambda)=\frac{2.4012\lambda+1.7597\lambda^2+2.3021\lambda^3}{1+3.7704\lambda+3.2250\lambda^2+2.3021\lambda^3}$$

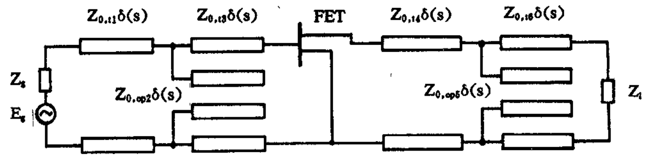

Fig. 2. Amplifier circuit for the 7–14 GHz range.

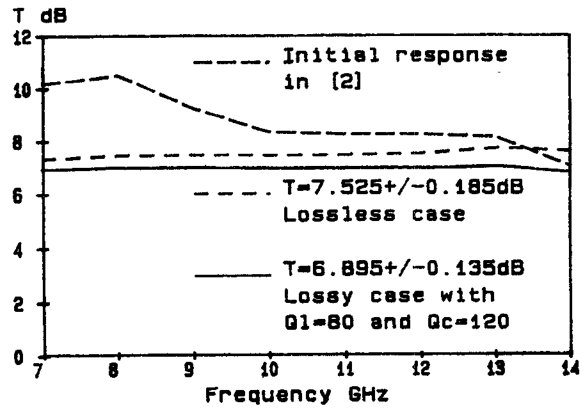

Fig. 3. Frequency response of the amplifier in Example 1.

TABLE I
AMPLIFIER PARAMETERS IN FIG. 2

Parameter	Impedance	Length
$Z_{0,t1}$	41.628 Ω	2.597 mm
$Z_{0,op2}$	39.741 Ω	2.597 mm
$Z_{0,t3}$	58.001 Ω	2.597 mm
$Z_{0,t4}$	218.549 Ω	3.037 mm
$Z_{0,op5}$	85.469 Ω	3.037 mm
$Z_{0,t6}$	90.030 Ω	3.037 mm

and their topologies can be synthesized as shown in Fig. 2. Line impedances and lengths are presented in Table I.

Step 5: Check whether the synthesized topologies are practical or not. If not, Kuroda's identities can be applied to shift those stubs so that each pair of stubs is spatially separated by a length of commensurate line. As for an ideal transformer, which may be encountered in the band-pass case for transforming the impedance level, the Norton transformation or partial stub extraction [14] (i.e., high-pass Kuroda identities) can be employed. In this example, the topologies of the input and output matching networks shown in Fig. 2 are directly achieved by alternately removing a cascaded line, followed by a shunt open-circuited stub, then a cascaded line.

Example 2

According to steps similar to those mentioned above, another one-stage broad-band FET amplifier for MMIC's is designed in which FET's S-parameter data are taken from [3]. The necessary inputs for the design are given as follows:

- Source impedance: $Z_g = 50$ Ω;
- Load impedance: $Z_l = 50$ Ω;
- Quality factor of the conductor: $Q_l = 100$;
- Quality factor of the dielectric: $Q_c = 150$;
- Frequency at which the above quality factors are measured: $f_m = 8$ GHz;

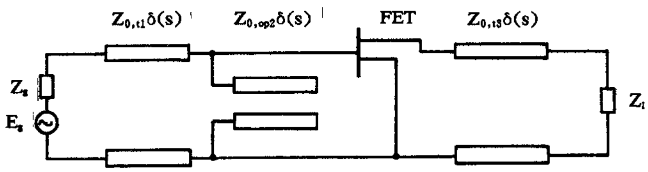

Fig. 4. Amplifier circuit for the 4–8 GHz range.

TABLE II
AMPLIFIER PARAMETERS IN FIG. 4

Parameter	Impedance	Length
$Z_{0,t1}$	49.118 Ω	6.186 mm
$Z_{0,\mathrm{op}2}$	58.061 Ω	6.186 mm
$Z_{0,t3}$	41.915 Ω	2.190 mm

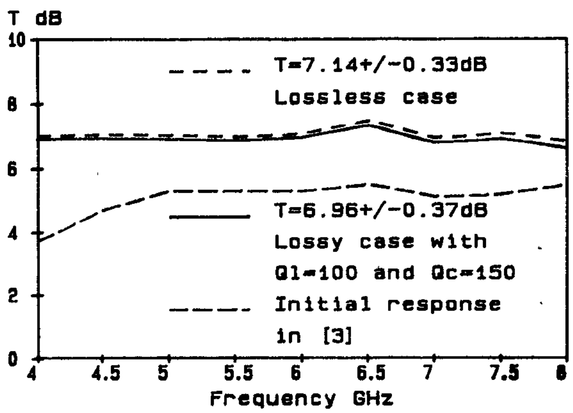

Fig. 5. Frequency response of the amplifier in Example 2.

- Passband: 4 GHz $\leqslant f \leqslant$ 8 GHz;
- Maximum complexity of the matching networks:
  input matching network: $n = 2$, $k = 0$, $m = 1$;
  output matching network: $n = 2$, $k = 0$, $m = 1$;
- Flat gain level to be approached: $T_0(\omega) = 7$ dB.

After optimization, it is found that the low-pass stub in the output matching network can be omitted. Thus, the final S parameters of the assumed lumped lossless input and output matching networks are as follows:

$$\tilde{e}_{11,1}(\lambda) = \frac{-0.4484\lambda + 0.423\lambda^2}{1 + 1.4308\lambda + 0.423\lambda^2}$$

$$\tilde{e}_{11,2}(\lambda) = \frac{-0.1773\lambda}{1 + 1.0156\lambda}.$$

The topology of the amplifier is shown in Fig. 4, and its parameters are given in Table II. The gain responses of the amplifier in lossy and lossless cases are shown in Fig. 5.

IV. CONCLUSIONS

The method described in this paper extends the real frequency techniques [6], [7] to the distributed lossy case and is straightforward in comparison with the techniques presented in [2] and [3]. It can handle lossy UE's and stubs with arbitrary frequency-dependent propagation constants and characteristic impedances and has all of the advantages of the real frequency techniques. Moreover, from the gain performances shown in Fig. 3 and Fig. 5, it can be seen that the optimized lossy gains are on the average about 0.63 dB and 0.18 dB lower than the gains calculated by taking the lossy elements out or, equivalently, by letting Q_l and Q_c be infinite. In practice, the lossy parameters, Q_l and Q_c, may vary over a wide range, so that it is worthwhile to take the losses of the lines into consideration in the synthesis of a lossy commensurate line network. It has also been shown that the ripples of the lossless gains are much less than those of the initial gain responses presented in [2] and [3].

APPENDIX
PROOF OF THE THEOREM

Assume that there are two impedances, given below, which satisfy the condition of the theorem:

$$z_1 = Z_2 f_1(Z_1/Z_2) \qquad \text{(A1a)}$$

$$z_2 = Z_2 f_2(Z_1/Z_2). \qquad \text{(A1b)}$$

Then, it is easy to verify that the parallel and series impedances between (A1a) and (A1b),

$$z_{3,p} = \frac{z_1 z_2}{z_1 + z_2} = Z_2 f_{3,p}(Z_1/Z_2) \qquad \text{(A2a)}$$

and

$$z_{3,s} = z_1 + z_2 = Z_2 f_{3,s}(Z_1/Z_2) \qquad \text{(A2b)}$$

have expressions similar to those of (A1), where $f_{3,p}$ and $f_{3,s}$ are still rational functions of Z_1/Z_2 as f_1 and f_2.

Suppose that at any port the input impedance of an n-port network built by the elements which satisfy the condition of the theorem has an expression similar to (A1),

$$z_i = Z_2 f_i(Z_1/Z_2) \qquad \text{(A3)}$$

where i $(i = 1, 2, \cdots, n)$ stands for the ith port. It can be clearly seen by calculations similar to (A2) that regardless of whether z_i is in parallel to or in series with z_1 in (A1a), the resulting impedance will have the form given in the theorem, i.e.,

$$z_{i,p} = Z_2 f_{i,p}(Z_1/Z_2) \qquad \text{(A4a)}$$

or

$$Z_{i,s} = Z_2 f_{i,s}(Z_1/Z_2). \qquad \text{(A4b)}$$

Thus, it can be concluded that if each element of an n-port network satisfies the condition of the theorem, any open-circuit impedance z_{ij} between port i and j $(i, j = 1, 2, \cdots, n)$ will meet the condition too. Therefore, in terms of the transformation of the theorem in [1], an expanded form of the transformation between the impedances of a lossy commensurate line network and a lumped lossless network is obtained. So are their impedance matrices written as (7) by means of the corollary in [1]. This completes the proof.

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